NUMERICAL INVESTIGATION OF THE FORMULAE
FOR THE ELIMINATION OF
GEOCENTRIC AND BARYCENTRIC PARALLAX
IN
PROF. LEUSCHNER'S SHORT METHOD.

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This Thesis Submitted in Partial Fulfillment
of the Requirements for the Degree
of
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Professor Leuschner has made a theoretical investigation[1] of the effect of parallax on an orbit and has given formulae for the elimination of the parallax which are independent of the geocentric distances. The purpose of this investigation is to make numerical tests of these formulae.

The parallax to be considered is twofold: geocentric and barycentric. Geocentric parallax is the angle at the body subtended by the radius of the earth drawn to the point of observation. Barycentric parallax is the angle at the body subtended by the distance from the center of the earth to the center of mass of earth and moon. The barycentric place enters into the problem by virtue of the fact that the equations of apparent motion of the Sun

$$X'' = -\frac{X}{R^3}; \quad Y'' = -\frac{Y}{R^3}; \quad Z'' = -\frac{Z}{R^3}$$

by means of which X" Y" and Z" are eliminated from

$$\xi'' = x'' + X''; \quad \eta'' = y'' + Y''; \quad \zeta'' = z'' + Z''$$

1): Pub. of L. O., Vol. Vll.

~~must be~~ referred to the center of mass of the earth and moon as origin ~~in order that the problem may be a two-body problem.~~ so that ξ'', η'', ζ'' must also be referred to this centre of mass. Both of these parallaxes consist of two parts - a systematic and a variable part. The variable part is equal to the differences of the parallax corrections for the three dates of observation.

For a partial elimination of parallax only the systematic part is eliminated by applying the following corrections to the Solar Coordinates for the Middle Date:

$$\Delta X = p_\alpha \Delta \cos\delta \sin\alpha + p_\delta \Delta \cos\alpha \sin\delta$$

$$(a) \quad \Delta Y = p_\delta \Delta \sin\alpha \sin\delta - p_\alpha \Delta \cos\delta \cos\alpha$$

$$\Delta Z = - p_\delta \Delta \cos\delta$$

For a complete elimination of the parallax two corrections are applied to the solar velocities in addition to the corrections given by (a). The first correction is $\Delta X_0', \Delta Y_0', \Delta Z_0'$ which is obtained from the $\Delta X, \Delta Y, \Delta Z$ of the three dates by a formula similar to that given for α_0' in the Publications of the Lick Observatory, Vol. V11.

$$\Delta X_0' = \frac{\Delta X_1'(t_0 - t_1) + \Delta X_{111}'(t_{111} - t_0)}{t_{111} - t_1}$$

$$(b) \quad \Delta Y_0' = \frac{\Delta Y_1'(t_0 - t_1) + \Delta Y_{111}'(t_{111} - t_0)}{t_{111} - t_1}$$

$$\Delta Z_0' = \frac{\Delta Z_1'(t_0 - t_1) + \Delta Y_{111}'(t_{111} - t_0)}{t_{111} - t_1}$$

~~and is~~ referred to the center of mass of the earth and moon as origin ~~in order that the problem may be a two-body problem~~. so that ξ'', η'', ζ'' must also be referred to this center of mass

Both of these parallaxes consist of two parts - a systematic and a variable part. The variable part is equal to the differences of the parallax corrections for the three dates of observation.

For a partial elimination of parallax only the systematic part is eliminated by applying the following corrections to the Solar Coordinates for the Middle Date:

$$\Delta X = f_\alpha \Delta \cos\delta \sin\alpha + f_\delta \Delta \cos\alpha \sin\delta$$

$$\Delta Y = f_\delta \Delta \sin\alpha \sin\delta - f_\alpha \Delta \cos\delta \cos\alpha \qquad (a)$$

$$\Delta Z = -f_\delta \Delta \cos\delta$$

For a complete elimination of the parallax two corrections are applied to the solar velocities in addition to the corrections given by (a). The first correction is $\Delta X_0', \Delta Y_0', \Delta Z_0'$ which is obtained from the $\Delta X, \Delta Y, \Delta Z$ of the three dates by a formula similar to that given for x_0' in the Publications of the Lick Observatory, Vol. VII.

$$\Delta X_0' = \frac{\Delta X_I'(t_0 - t_I) + \Delta X_{III}'(t_{III} - t_0)}{t_{III} - t_I}$$

$$\Delta Y_0' = \frac{\Delta Y_I'(t_0 - t_I) + \Delta Y_{III}'(t_{III} - t_0)}{t_{III} - t_I} \qquad (b)$$

$$\Delta Z_0' = \frac{\Delta Z_I'(t_0 - t_I) + \Delta Z_{III}'(t_{III} - t_0)}{t_{III} - t_I}$$

The second correction to be applied to the solar velocities is one involving a quantity β which is defined as follows:

$$\beta = \frac{-\frac{s}{2} \sin (a - \alpha_{11}) \left[1 - \frac{\tan P}{\tan P'} \right]}{\alpha_0' - \cos \delta \tan P \tan \delta_0'}$$

s, a, P and P' are auxiliary quantities depending on the solar coordinates and their corrections, the comet's right ascension and declination and the intervals. This correction depends, in the main, upon the second differences of the $\Delta X, \Delta Y, \Delta Z$ and is necessary to refer ~~the expression (a)~~ ξ'', η'', ζ'' to the center of the earth. The corrected solar velocities become

$$(\alpha) \quad \begin{aligned} (X_0') &= X_0' + \Delta X_0' + \beta \cos \alpha_{11} \\ (Y_0') &= Y_0' + \Delta Y_0' + \beta \sin \alpha_{11} \\ (Z_0') &= Z_0' + \Delta Z_0' + \beta \tan \delta_{11} \end{aligned}$$

The solar coordinates and velocities used above refer to the center of the earth as origin. Corrections must also be applied to the solar coordinates and velocities in order that they may be referred to the center of mass as origin. Let $\Delta_1 X, \Delta_1 Y, \Delta_1 Z$ be the corrections necessary for elimination of the geocentric parallax of the observed body (formula (a)); $\Delta_2 X, \Delta_2 Y, \Delta_2 Z$ for the barycentric parallax of the body. These

are also obtained from formulae (a) with the exception that the barycentric parallax factors are used. These factors are

$$p_\alpha^m \rho = d_1 \frac{\cos\delta_1}{\cos\delta} \sin(\alpha - \alpha_1)$$

$$p_\delta^m \rho = d_1 \{ \sin\delta \cos\delta_1 \cos(\alpha - \alpha_1) - \cos\delta \sin\delta_1 \}$$

These formulae are derived from the formulae for any correction to α and δ *. α_1 and δ_1 are the moon's right ascension and declination at the dates of the observations.

$$\Delta_2 X = p_\alpha^m \rho \cos\delta \sin\alpha + p_\delta^m \rho \cos\alpha \sin\delta$$

$$(c) \qquad \Delta_2 Y = p_\delta^m \rho \sin\alpha \sin\delta - p_\alpha^m \rho \cos\delta \cos a$$

$$\Delta_2 Z = -p_\delta^m \rho \cos\delta$$

Corrections $\Delta_3 X, \Delta_3 Y, \Delta_3 Z$ for barycentric parallax must also be applied to the Solar coordinates. The Solar coordinates X, Y and Z referred to the center of mass as origin are equal to the coordinates referred to the center of earth plus the coordinates of the center of the earth referred to the center of mass. Let d be the distance of the center of mass from the center of the earth. Let α_1 and δ_1 be the coordinates of the moon and as the center of mass lies in the line joining the center of the moon with the center of the earth,

*Publ. L. O. Vol. V11, part 7, page 11, formulae (15).

the rectangular geocentric coordinates of the center of mass are

$$(f)\qquad \begin{aligned} x &= d_1 \cos\delta_1 \cos\alpha_1 \\ y &= d_1 \cos\delta_1 \sin\alpha_1 \\ z &= d_1 \sin\delta_1 \end{aligned}$$

The coordinates of the center of the earth referred to the center of mass as origin are equal to (f) with opposite signs. Therefore

$$(g)\qquad \begin{aligned} \Delta_3 X &= -d_1 \cos\delta_1 \cos\alpha_1 \\ \Delta_3 Y &= -d_1 \cos\delta_1 \sin\alpha_1 \\ \Delta_3 Z &= -d_1 \sin\delta_1 \end{aligned}$$

The value of the distance d_1 is obtained from

$$d_1 = \frac{\pi}{\pi_1} \frac{\mu_1}{1+\mu_1}$$

where π and π_1 are the mean equatorial horizontal parallaxes of the sun and moon. $\mu_1 = \frac{m}{m} = \frac{\text{mass of the moon}}{\text{mass of the earth}}$. Therefore $d_1 = 6''.4373$.

The corrected solar coordinates corresponding to the elimination of the geocentric and barycentric parallax in the coordinates become

$$(h)\qquad \begin{aligned} (X) &= X + \Delta_1 X + \Delta_2 X + \Delta_3 X \\ (Y) &= Y + \Delta_1 Y + \Delta_2 Y + \Delta_3 Y \\ (Z) &= Z + \Delta_1 Z + \Delta_2 Z + \Delta_3 Z \end{aligned}$$

In order that the solar velocities be referred to the center of mass corrections $\Delta_3 X_0', \Delta_3 Y_0', \Delta_3 Z_0'$ derived from formulae (b) using $\Delta_3 X, \Delta_3 Y, \Delta_3 Z$ from (g) for the three dates are applied.

It is evident of course that the complete elimination of geocentric and barycentric parallax should give the same results as if the observations had been corrected beforehand for parallax on the basis of the known value of ρ (the geocentric distance) and the proper parallax factors.

In order to test the effect of the parallax corrections on an orbit and in order to test the formulae for the elimination of parallax the following orbits have been computed and compared. All of these orbits are based on the same three observations of comet a 1909 (Daniel).

	1909 Gr. M. T.	α (1909.0)	δ (1909.0)
1.	June 16.5306	25° 28' 38".00	+ 29° 58' 25".00
11.	June 18.9809	27 12 29 .00	+ 33 26 22 .00
111.	June 21.9659	29 27 51 .00	+ 37 25 17 .00

The computation has been carried only to the heliocentric coordinates and velocities, these being the first comparable quantities in the orbits.

Orbit 1 - Geocentric parallax partially eliminated by correcting Solar coordinates of the middle place - computed by Dr. Crawford.

Orbit 11 - Geocentric parallax fully determined using Dr.

Crawford's parallax factors and geocentric distances.

Orbit 111 - Geocentric parallax completely eliminated by the formulae.

Orbit 1V - Parallax entirely neglected.

Orbit V - Geocentric and barycentric parallax completely eliminated by the formulae.

Orbit V1 - Observations corrected for geocentric and barycentric parallax.

Details of the computation:-

Orbit 1 - This orbit was computed by Professor Crawford and published in Lick Observatory Bulletin Number 159. The right ascensions and declinations were used as given above without change. The Solar coordinates for all three dates and the Solar velocities for the middle date were interpolated from the Nautical Almanac -

		X	Y	Z
1	June 16.5306	+ 0.0854344	+ 0.9288751	+ 0.4029449
11	June 18.9809	+ 0.0440423	+ 0.9314663	+ 0.4040708
111	June 21.9659	- 0.0064722	+ 0.9324818	+ 0.4045127
		X'_o	Y'_o	Z'_o
11	June 18.9809	9.992630_n	8.630115	8.268346

The geocentric parallax is partially eliminated for the second dates by formulae (a).

$\Delta X_{\circ}$ - 0.0000073 $\Delta Y_{\circ}$ + 0.0000320 $\Delta Z_{\circ}$ - 0.0000123

These corrections are applied to X, Y and Z of the middle place. Then:

$X_{\circ}$ + 0.0440350 $Y_{\circ}$ + 0.9314983 $Z_{\circ}$ + 0.4040585

Orbit 11 - The right ascensions and declinations at the three dates were corrected for geocentric parallax on the basis of the geocentric distances derived in Orbit 1. The parallax corrections are as follows:-

	1	11	111
p_{α}''	- 7".78	- 8".18	- 8."65
p_{δ}''	+ 5 .96	+ 3 .18	+ 3 .14

The corrected observations are:-

	1	11	111
α	25° 28' 30".22	27° 12' 20".82	29° 27' 42".35
δ	+ 29 58 30 .96	+33 26 25 .18	+37 25 20 .14

These were made the basis of the calculation together with the Solar Coordinates X, Y, Z as interpolated from the Ephemeris.

Orbit 111 - Based on the same values of α and δ as in Orbit 1. The Solar coordinates for the middle date corrected as in Orbit 1 in addition $\Delta_1 X$, $\Delta_1 Y$, $\Delta_1 Z$ were computed for the first and third dates. The three sets of corrections are as follows:

$\Delta_1 X_1$ -	0.0000010	$\Delta_1 Y_1$ +	0.0000335	$\Delta_1 Z_1$ -	0.0000235
$\Delta_1 X_0$ -	73	$\Delta_1 Y_0$ +	320	$\Delta_1 Z_0$ -	123
$\Delta_1 X_m$ -	82	$\Delta_1 Y_m$ +	332	$\Delta_1 Z_m$ -	120

From thse their velocities by formulae (b) are computed:

$\log \Delta X_0'$ 5.9542_n $\log \Delta Y_0'$ 4.9547_n $\log \Delta Z_0'$ 6.1719

In addition β was computed by formula (c). The resulting values of X_0', Y_0' and Z_0' from (d) are as follows:

X_0' 9.992280_n Y_0' 8.634617 Z_0' 8.286768

Orbit lV - The right ascensions and declinations as in Orbit l and the Solar Coordinates uncorrected.

Orbit V - The right ascensions and declinations are taken as in Orbit l. The solar coordinates for the three dates are corrected for their barycentric parallax and the geocentric and barycentric parallaxes in the observed places eliminated by (h). The corrections applied to the Solar Coordinates are:

l.

$\Delta_1 X_1$ -	0.0000010	$\Delta_1 Y_1$ +	0.0000335	$\Delta_1 Z_1$ -	0.0000235
$\Delta_2 X_1$ -	91	$\Delta_2 Y_1$ -	189	$\Delta_2 Z_1$ -	2
$\Delta_3 X_1$ -	90	$\Delta_3 Y_1$ -	275	$\Delta_3 Z_1$ +	117

ll.

$\Delta_1 X_0$ -	73	$\Delta_1 Y_0$ +	320	$\Delta_1 Z_0$ -	123
$\Delta_2 X_0$ -	168	$\Delta_2 Y_0$ -	269	$\Delta_2 Z_0$ -	71
$\Delta_3 X_0$ +	86	$\Delta_3 Y_0$ -	269	$\Delta_3 Z_0$ +	132

111.

$\Delta_1 X_{111}$ - 0.0000082	$\Delta_1 Y_{111}$ + 0.0000332	$\Delta_1 Z_{111}$ - 0.0000120
$\Delta_2 X_{111}$ - 214	$\Delta_2 Y_{111}$ - 276	$\Delta_2 Z_{111}$ - 129
$\Delta_3 X_{111}$ + 251	$\Delta_3 Y_{111}$ - 158	$\Delta_3 Z_{111}$ + 97

The solar coordinates become:

X_1 + 0.0854153	Y_1 + 0.9288622	Z_1 + 0.4029329
X_0 + 0.0440268	Y_0 + 0.9314445	Z_0 + 0.4040646
X_{111} - 0.0064767	Y_{111} + 0.9324716	Z_{111} + 0.4044975

The solar velocities are referred to barycentric place by means of the $\Delta_3 X, \Delta_3 Y, \Delta_3 Z$ given above. Using formula (b).

log $\Delta X'_0$ 6.5731	log $\Delta Y'_0$ 6.0223	log $\Delta Z'_0$ 5.0486$_n$

The Solar velocities which are referred to the center of mass arë corrected for geocentric and barycentric parallax using formula (d). applied $\Delta_1 X, \Delta_2 X; \Delta_1 Y, \Delta_2 Y$ etc

log $\Delta X'_0$ 6.3630$_n$	log $\Delta Y'_0$ 6.0770$_n$	log $\Delta Z'_0$ 4.8886
log $\rho \cos\alpha_n$ 6.9824	log $\rho \sin\alpha_n$ 6.6934	log $\beta \tan\delta_n$ 6.8531

The solar velocities are as follows:

(X'_0) 9.992142$_n$	(Y'_0) 8.634969	(Z'_0) 8.284648

Orbit Vl - The right ascensions and declinations at the three dates are corrected for geocentric and barycentric parallax on the basis of the geocentric distances derived in Orbit 1. The parallax corrections are as follows:

	1.	11.	111.
ρ_α''	- 13".08	- 15".36	- 15".51
ρ_δ''	+ 5 .92	+ 1. 37	+ 0 .25

The corrected observations are

	1.	11.	111.
α	25° 28' 24".92	27° 12' 13".64	29° 27' 35".49
δ	+ 29 58 30 .92	+ 33 26 23 .37	+ 37 25 16 .75

The solar coordinates are referred to barycentric place by applying the corrections

$\Delta X_\odot + 0.0000086$ $\Delta Y_\odot - 0.0000269$ $\Delta Z_\odot + 0.0000132$

The solar coordinates are as follows:

$X_\odot + 0.0440509$ $Y_\odot + 0.9314394$ $Z_\odot + 0.4040840$

The solar velocities are referred to barycentric place by appluing the corrections

$\log \Delta X_\odot'$ 6.5731 $\log \Delta Y_\odot'$ 6.0223 $\log \Delta Z_\odot'$ 5.0486

The solar velocities then become

$\log X_\odot'$ 9.992465_n $\log Y_\odot'$ 8.631185 $\log Z_\odot'$ 8.268084

The following values of the comet's heliocentric coordinates and velocities are obtained for the different orbits.

In numbers.

	1.	11.	111.	1V.	V.	V1.
X_0	+0.668778	+0.669700	+0.669271	+0.668780	+0.669562	+0.669853
Y_0	-0.565036	-0.564561	-0.564782	-0.565000	-0.564584	-0.564483
Z_0	+0.125224	+0.125908	+0.125589	+0.125217	+0.125793	+0.125997
X_0'	+0.446200	+0.443864	+0.445037	+0.446198	-0.443721	+0.443432
Y_0'	+0.347541	+0.347166	+0.347367	+0.347548	+0.347165	+0.347085
Z_0'	+1.393368	+1.393785	+1.393542	+1.393367	+1.393897	+1.393841

The following comparisons are of particular interest.

	V1-V	V1-1V	1-1V
X_0	+ 291	+ 1073	- 2
Y_0	+ 101	+ 517	+ 36
Z_0	+ 204	+ 780	+ 7
X_0'	- 289	- 2766	+ 2
Y_0'	- 85	- 463	- 7
Z_0'	+ 216	+ 454	- 19

The differences between Orbit 1V and Orbit V1 show that Orbit 1V is correct only to two places. In the comparison of Orbit 1 and Orbit 1V where the parallax is entirely neglected the differences are so small and at the same time are within the errors of computation that it is quite evident that the partial elimination has no decided effect upon the orbit.

The differences between Orbit V and V1 where parallax

is completely eliminated show that Orbit Vl is correct to three places. Orbit V, depending on differential relations, is more accurate than Orbit Vl. Therefore the elimination formulae for parallax give an orbit accurate to two more places than the omission of parallax. The gain of two places however is only apparent if the errors of observation are comparable with the variable part of the parallax corrections. The differences between Orbit 11 and Orbit 111, which should check, are, ~~however,~~ within the errors of the computation except in the case of X' where the largeness of the difference may arise from an accumulated error.

The differences between Orbit V (geocentric and barycentric parallax eliminated) and Orbit Vl (geocentric and barycentric parallax fully determined) are of the greatest importance and interest. The differences are within the error of computation and it is seen from them that Orbit Vl is correct to four places. This is a gain of two places over Orbit lV. In comparing the differences between Orbit Vl and Orbit lV with the differences between Orbit V and Orbit Vl it is seen that ~~practically~~ all of the parallax corrections have been taken into account and ~~those~~ the differences that remain are within the error of the computation. Therefore for corrt a 1909 Orbit V is the most

accurate. The gain in accuracy of two places justifies the added work in the computation.

Although the formulae for the elimination of barycentric and geocentric parallax should be used in comet a 1909, the question presents itself whether this method should be pursued in all cases. As stated at the beginning there is a systematic and a variable part in parallax. The systematic part which alone was taken care of in Orbit 1 gave but a small correction; the variable part which is the difference between the parallaxes for the different dates of observation and hence the velocities in the parallaxes gives the largest corrections, as is shown in Orbit V. It is evident therefore that if the variable part in the parallaxes is not greater than the error of the observation the corrections would have no value, for if they were smaller the observations would be corrected by a correction to more places than they themselves can be relied upon. The differences in the parallax are greater than the errors of observation only when the comet is rapid moving and fairly near the sun. Then the method of Orbit V is to be pursued. In all other cases the parallax is to be entirely neglected.

UNIVERSITY OF CALIFORNIA
BERKELEY ASTRONOMICAL DEPARTMENT
(STUDENTS' OBSERVATORY)

BERKELEY, CAL., Dec 4. 1909.

Miss Sarah de Camp Morgan
Students' Observatory.

Dear Miss Morgan,

Orbit V is not quite correct. It will be necessary to repeat part of the computation, namely ρ and all the quantities depending on same. The equation of motion for the Sun is true in the form

$$X'' = \frac{X+\Delta X_3}{(R+\Delta R_3)^3}$$

But we use

$$X'' = \frac{X+\Delta X_1+\Delta X_2+\Delta X_3}{(R+\Delta R_1+\Delta R_2+\Delta R_3)^3} = \frac{(X)}{(R)^3}$$

which is therefore in error only by $\Delta X_1+\Delta X_2$, and $\Delta R_1+\Delta R_2$. The error arising from $\Delta R_1+\Delta R_2$ is ineffective as it disappears in the ratio $\frac{\sigma'}{\sigma}$. The error $\Delta X_1+\Delta X_2$ is taken care of in the term $\frac{1}{(R)^3}(\Delta X_1+\Delta X_2)$ of

$$\left(\frac{1}{(R)^3}+\frac{d_x}{\theta_1\theta_{III}}\right)(\Delta X_1+\Delta X_2)$$

You have used (ΔX) in this formula $= \Delta X_1+\Delta X_2+\Delta X_3$.
You should have used $(\Delta X) = \Delta X_1+\Delta X_2$.

UNIVERSITY OF CALIFORNIA
BERKELEY ASTRONOMICAL DEPARTMENT
(STUDENTS' OBSERVATORY)

BERKELEY, CAL., Oct. 4, 1909

Miss Sarah de Caney Morgan
Students' Observatory

Dear Miss Morgan:

Orbit V is not quite correct. It will be necessary to repeat part of the computation, namely D and all the quantities depending on it. The equation of condition for the Sun is then in the form

$$X'' = \frac{X+\Delta X_3}{(R+\Delta R_3)^3}$$

but not as

$$X'' = \frac{X+\Delta X_1+\Delta X_2+\Delta X_3}{(R+\Delta R_1+\Delta R_2+\Delta R_3)^3} = \frac{(X)}{(R)^3}$$

which is changed in error only by $\Delta X_1+\Delta X_2$ and $\Delta R_1+\Delta R_2$. The error arising from $\Delta R_1+\Delta R_2$ is sufficient as it also appears in the ratio G'. The error $\Delta X_1+\Delta X_2$ is however one of in the term $\frac{1}{R^3}(\Delta X_1+\Delta X_2)$ of

$$\left(\frac{1}{R^3}+\frac{dx}{\rho_0 \rho_m}\right)(\Delta X_1+\Delta X_2)$$

You have used (ΔX) in the formulas $= \Delta X_1+\Delta X_2+\Delta X_3$. You should have used $(\Delta X) = \Delta X_1+\Delta X_2$.

Miss Morgan – 2 –

UNIVERSITY OF CALIFORNIA
BERKELEY ASTRONOMICAL DEPARTMENT
(STUDENTS' OBSERVATORY)

BERKELEY, CAL.,

Similarly d_x, d_y, d_z should have been computed from the $\Delta X = \Delta X_1 + \Delta X_2$, etc. In ~~order~~ other words ΔX_3 does not enter into ~~[illegible]~~ ~~into~~ d_x, d_y, d_z, s, a, d, or β. With the exception of this error in β the first work on orbit V is correct ~~save that you~~ In the second orbit you ~~have~~ (page 5a, etc) you have introduced ΔX_3 twice. In computing the $(\Delta X)'_0$ from the $(\Delta X) = \Delta X_1 + \Delta X_2 + \Delta X_3$, the $(\Delta X)_3$ is included, so that you should not have revised your work by introducing it a second time into the solar velocities. For $(X)'_0$ we get then

$$(X)'_0 = X'_0 + (\Delta X_1 + \Delta X_2 + \Delta X_3)'_0 + \beta \cos\alpha \text{, etc.}$$

Your only error is then in the β

Can you rectify the error by Monday, 2 p.m.?

Orbit VI appears to be correct.

Very sincerely yours,

A. O. Leuschner

☌ a 1909

Kobold	Crawford	Kobold
June 16, 17, 18	June 16 18 21	16, 20, 24
T 1909 June 5.30 B.M.T	June 5.17 Gr.M.T	5.30 B.M.T. 5.26 Gr.M.T.
ω 5° 3.5'4	4° 59'	5° 0.1'
☊ 305 21.32	306 19	305 38
i 51. 53.62	52 26	52 4
log q 9.92524		9.92583
q 0.842	0.846	0.843

Boss

16, 17, 18 corfu 21

T 5.12 Gr.M.T.

ω 4° 56'

☊ 306 40

i 52 39

log q 9.92815

q 0.848

	Kobold II - Cr.	Kobold II - Boss
ΔT	+ .09 d	+.14 d
Δω	+ 2'	+ 5'
Δ☊	− 41	− 62
Δi	− 22	− 35
Δq	− .003	− .005

Kobold II - I

ΔT − .05

Δω − 2.5'

Δ☊ + 17

Δi + 10

Δq [illegible]